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ABSTRACT

The HeadJam series is comprised of six programs exploring mathematics, science, and critical thinking skills. It is an award-winning, educational videotape series for middle school students that explores multi-disciplinary skills in a highly entertaining way. The teacher's guide and 22-minute video, "Go Figure," demonstrate how math is used in the real world, where problem solving happens in unlikely places. They also look at how math impacts every facet of students' lives from music to video games to roller coaster rides. Related classroom activities are also included. (ASK)

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ED 460 837

HeadJam
Go Figure

Teaching Guide

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HEADJAM

go figure

Teaching Guide

Video Game University:

Future Programmers' Paradise

Several years ago, a national magazine ran a cartoon captioned, "A Parent's Fantasy." The drawing depicted several newspaper classified ads, all offering employment at top wages for video game players. The implication, though humorous, was that parents who thought their child's video game-playing skills might lead to any kind of a career were dreaming.

How times have changed.

In February 1998, DigiPen Institute of Technology opened in Redmond, Wash. The school offers the first ever baccalaureate degree of science in Real Time Interactive Simulation, which is an academic way of saying that students can now major in video game programming.

The degree is an intensive four-year program, which focuses on video game programming and computer animation with emphasis on real-time interactive simulations. Students learn the skills needed for the rapidly growing, high-tech video game and special effects industries. And possessing those skills can bring in a healthy paycheck.

There were 40 students in the initial class, but DigiPen now accepts 100 students per year for the Bachelor of Science program and will accept the same number for its inaugural Bachelor of Arts program class in 2003. Not bad for a school whose first students learned about it reading *Nintendo Power Magazine*.

Birth of DigiPen

Claude Comair, the school's founder, began the program with a two-year course at a facility in

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*Articles and classroom activities are designed to complement the video, *Go Figure*.

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Vancouver, B.C., in 1994. He approached the Nintendo corporation for sponsorship, and the company, foreseeing the need for future programmers (as well as trying to fill current openings) supported the first school with guidance, technical expertise, and donations of hardware and development tools. They continue the support with the new school in Washington state.

The success of the Vancouver program (without advertising, the school received 12,000 applications for 80 openings) led Comair to expand the curriculum to offer the bachelor's degree.

Students aren't automatically offered positions with Nintendo, nor are they required to work for the corporation when they graduate. But finding a job seems to be no problem (a third of one class at the Vancouver school was lured away by other game manufacturers before graduation).

"It is of utmost importance that our students love what they do, and the technical skills that we teach will enable them to get good jobs in the private sector," says Comair. "The demand for trained talent in these fields is so great that our students weigh offers from the major players here in the Northwest and Hollywood in the video game design and computer animation fields well before they complete the curriculum."

$15 Billion Industry

Video games have become a $15 billion industry worldwide, and that's just on consoles alone. Add in games played on home computers, the Internet, and in arcades, and the figures grow even more. Nearly half of all homes in the U.S. have at least one video game console. So, it's a sure bet that the market will continue to grow for the games and for the people who can design and produce increasingly sophisticated hardware and software.

Not Your Typical Campus

While DigiPen Institute is an accredited program, it's the high-tech flip side to a sprawling university. The classes are intense, year-round, and can last all day if the students are involved in a project.

The students come from a variety of backgrounds and educational levels. Some are right

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out of high school, while others range up to age 30 and have college degrees. What they share is imagination, creativity, and a real love of playing video games.

Brush Up Your Math and Science...

Comair describes the ideal DigiPen student as "a creative person who is also an unbelievable scientist." Since that isn't always the case, students get a heavy dose of math and science (if they haven't already had sufficient courses in high school or college–it's not a requirement to come to the institute with advanced math, since it is offered there).

Comair estimates that students spend "90 percent of their time learning general mathematics and computer graphics mathematics, which is more toward matrices, matrix algebra, vector geometry, and so forth. Math is at the core of programming and understanding computers."

Physics is also a vital field for the future programmers. "Most of the time, games are an interactive simulation of the real world," says Comair. "The real world is governed by the laws of physics–gravity, motion dynamics, wave optics and so on. And these have to be understood by the person who is trying to create that simulation."

For example, one group of DigiPen students designed a game that called for the hero to swing from place to place using his whip as a rope. The physics involved in rope swinging–tension, flexibility, motion–had to be studied for the movement of the character to look right.

The mechanics of programming are also a substantial part of the curriculum. Students must learn how a computer works, from binary system of ones and zeros that all programming uses, through computer languages.

...And Brush Up Your Painting

Once a designing student has the math and physics under his belt, he has to become a pointillist painter. That's because a video screen is really row upon row of pixels, or dots, with each identified in the computer's memory by a number.

Armed with the education and the imagination, graduates of DigiPen will be designing the games of the future. Right now, the market for those games is dominated by 6- to 14-year-old males (60 percent of game buyers fall in this demographic), but some game-company officials think a female market is growing. With the popularity of "Xena, Warrior Princess" on television, is a "Xena" video game, played by hoards of little (and probably some big) girls, far behind?

What the Target Market Thinks

Recently a group of 10 of the 6- to 14-years-old males gave their opinions on why video games are so popular. (Granted, it's a small sample, but it gives a glimpse into the mind of the inveterate video game jockey.)

When asked the most basic question–"Why do you like to play video games?"–all the boys responded the same way: "Because it's fun."

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It's all up in the Air

Have you ever juggled your schedule? Maybe you've heard of a shady accountant who juggled the books. If you're really stressed, you might say you're trying to keep all the balls in the air.

These are just a few modern metaphorical uses for an old art. Juggling is an activity that most people associate with circuses and magicians, but it can have applications in the classroom as well.

One Ball, Two Balls, Three Balls...

There's evidence that juggling (the manual kind) was going on in ancient Egypt, Greece, and Rome. In fact, the term comes from the Latin *joculare* meaning, "to jest." By medieval times in France, the word had become *jongleurs*. These were entertainers who combined music, storytelling, acrobatics, and recitations with juggling as they traveled the countryside, performing at festivals, fairs, and in marketplaces.

Eventually the craft of juggling spread to our shores. Traveling tent shows, circuses, and eventually vaudeville acts weren't complete without at least one juggler on the bill. In the 1950s, you could count on Ed Sullivan to bring plate spinners and knife tossers into your living room. And today the ancient art has gotten glitzy, featured in splashy Las Vegas shows and on Broadway.

Juggling Math, Science and Research

The *Go Figure* video accompanying this publication features a man juggling balls and demonstrating how the ball tosses correspond to time sequences. If he seems adept at keeping all those objects rhythmically floating, he should. He's Ron Graham, a man who juggles much more than balls.

Graham is chief scientist at AT&T Labs Research. He also teaches part-time at Rutgers University; lectures and presents seminars around the world; and is very active in the American Mathematical Society. In his spare time, he sits on the editorial board of 40 (you read that right) mathematics and computer journals. And he's past president of the International Jugglers Association.

Graham is interested, to say the least, in number theory and other areas of mathematics. His work in these fields earned him the prestigious Polya Prize in 1972 and membership in the National Academy of Sciences in 1985. He also served for two years on a high-profile National Research Council committee on cryptography, which recommended that the United States ease restrictions on encryption regulations.

Problem Solving

Graham is generous with one of the secrets of his success, and it's applicable to just about any area of life. "The best way to solve a complex problem is to break it down into component parts, learn each of the parts, and learn how the parts go together," he notes.

Math is connected to many of his activities. Juggling is a prime example. Many of the 3,000 members of the International Jugglers Association are interested or employed in mathematics or computers.

"It's difficult to explain this connection in any straightforward way, but...juggling combines abstract patterns and mind-body coordination in a pleasing way," Graham says.

A Matter of Math

As Graham demonstrates in *Go Figure*, he connects math and juggling through the pattern of the tosses or "site swaps." He shows a three-ball cascade (a juggling pattern) along a number line, drawing loops from number to number to show from where each ball leaves and to where each ball will fall. In the case of three balls, each ball will fall every third number.

Or as he puts it: "The three-ball cascade is perhaps the most basic juggling trick. Balls are thrown alternately from each hand and travel in a figure-eight pattern.

"This pattern has a natural generalization for any odd number of balls but can't be done in a natural way with an even number of balls. Even if simultaneous throws were allowed, in a symmetrical cascade with an even number of balls, there would be a collision at the center of the figure eight," says Graham.

Juggling to Keep Kids in School

Besides the practical applications with math, juggling has proved to be a winner with some students in unexpected ways. As one school discovered, tossing beanbags around can keep some at-risk kids involved.

Teachers at H. D. Perry Middle School in south Florida found that kinesthetic learners (the hands-on, physical types) usually were found in the school's dropout prevention program. Putting balls in these students' hands and challenging them with juggling soon followed. It turns out that most of the kids were naturals at the art, often excelling while the teachers struggled. It was a role reversal of expert and novice that changed everyone's perspective.

For three years, the faculty also used juggling as a service learning project. Students who formerly often were truant surprised the staff by staying late after school four to five days a week to practice. Some of the least confident learners became mentors to grade school children in math and juggling, making a difference in the lives of younger students.

CLASSROOM ACTIVITY

How to Juggle

Juggling takes skill, control, and a will to learn. Here are instructions for the basic three-ball juggle (the cascade) that may have you and your students juggling with abandon! Instead of balls, you might begin with beanbags or wadded up socks. They're easier to manage. Another tip–practicing a foot or two from a wall can help if your balls seem to be wandering forward and getting out of control. The balls can glance off the wall and you can continue without dropping the balls.

Juggling One Ball

First, learn with one ball. Throw the ball from hand to hand so it peaks at about eye level. The ball should be thrown from the palm of the hand and the hands should be at about hip level and on either side of the body. After you have the throw and catch perfected with one ball to and from both hands, move on to two balls.

Juggling Two Balls

Hold one ball in each hand. Throw the first ball (it doesn't matter which one) as you did in the first exercise but when it reaches its peak, throw the second ball. Catch the first and then the second. Practice this until you are comfortable with starting with either your left or right hand.

Juggling Three Balls

Now you go to three balls. Hold two in one hand (either hand, whichever is more comfortable) and one in the other. Start by practicing three throws. Throw a ball from the hand holding two balls (we'll say the right hand). As this first ball reaches its peak, throw the ball from your left hand. The first ball will be caught before the second reaches its peak. As the second ball peaks, throw the ball from your right hand. This is quite difficult, especially catching the balls. Don't be disheartened if it seems as though the balls spend more time on the ground than in the air. All you need is practice. When you can get three consecutive throws and catches, then add another throw. After four throws and catches, make it five and then just keep going. Before you know it, you'll be juggling.

Good Luck!

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FLYING CARPETS ON WHEELS

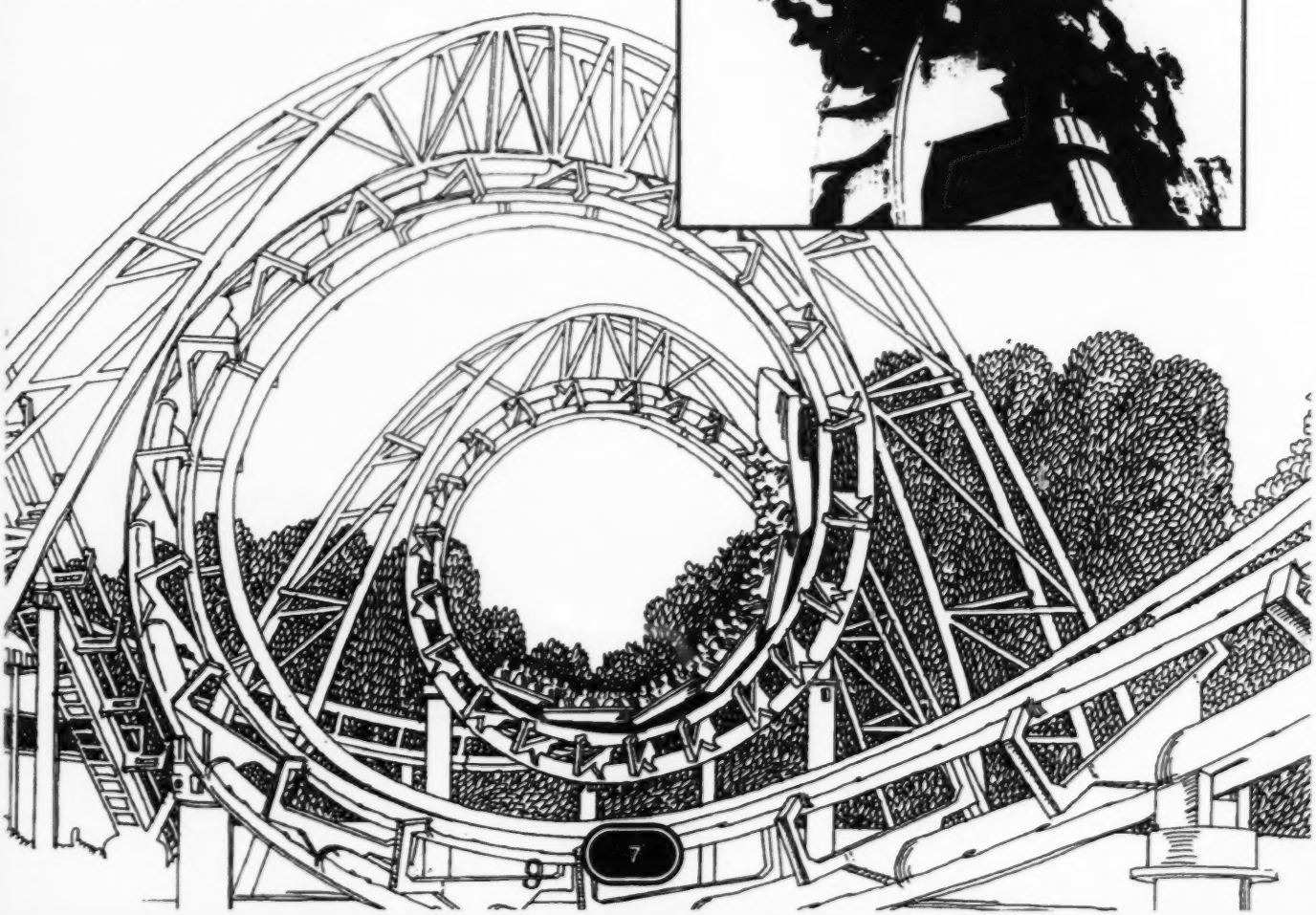
What is it about roller coasters that provokes someone to stand in line for three hours just to ride one? Some think it has to do with "Fight or Flight" syndrome. When a person gets scared, the pancreas produces large amounts of adrenaline that helps face fears instead of fleeing. While waiting in line, the body produces adrenaline. After the ride, an excess of adrenaline produces an "adrenaline rush." That's why some people notice they have an abundance of energy after stepping off a roller coaster.

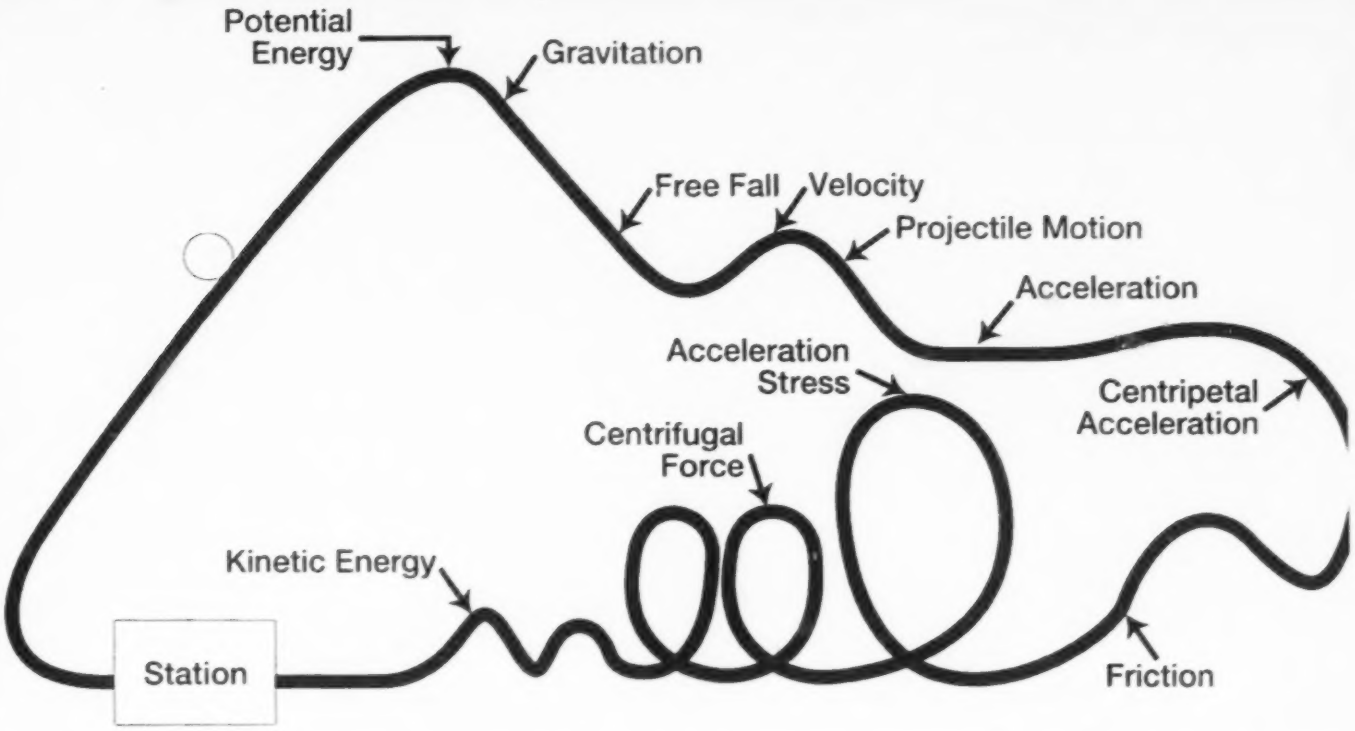
From Russia...

That people will actually pay to be scared was evident as far back as the 1400s when the earliest roller coasters were developed in Russia. Carts made of ice rode on ice slides built of wood or sometimes built into mountains. Some slides were 70 feet high. Large holes chiseled into the ice carts were filled with straw and used as seats. People climbed long sets of stairs to the top for a quick ride to the bottom.

...to France

Soon, these large slides began appearing in other countries with warmer climates, only the ice and straw were replaced with wood and steel. Many of these rides were constructed in France in the 1700s and were called Les Montagnes Russe, which means Russian Mountains.





America, at last!

The first roller coaster-type ride in America didn't appear until the 1800s when a passenger car was added to an old gravity-powered mine train. Soon the train was a full-time attraction. Mules pulled the carts up the hill and paying customers rode the carts to the bottom, reaching six miles per hour.

It's All Physics

Through the years, roller coasters evolved. Designers added more twists, turns, drops, and even loops. Yet there's one thing all coasters have in common: a fundamental basis on the principles of physics.

Aside from clothoid loops, the roller coaster is basically a simple machine, and can be easily thought of in terms of kinetic and potential energy. The coaster's lift motor exerts enough energy to lift the carts to the top of the hill, increasing their potential (stored) energy (PE). As the first cart reaches the peak, its PE is converted to kinetic energy (KE), the energy of motion. The process is repeated for each cart. KE increases as the coaster travels down the hill. But when it reaches the bottom and starts to climb the next hill, KE is again converted to PE.

Gee! G Forces!

Another important concept in roller coaster physics is g (gravity) forces. As the cart goes up and down and makes sharp turns, the passengers' bodies feel the same forces that astronauts feel while traveling into space. These forces are recorded by accelerometers and calibrated in g's. When standing on earth, people normally experience the sensation of 1 g of acceleration vertically, which equals an acceleration of 9.8 m/s2. A g force of 3 gives a person the sensation of being three times heavier than normal weight.

During a roller coaster ride, different positions cause different g forces. Positive g forces, which give the sensation of a weight increase, are experienced during ascent. Negative g forces are experienced during descent and give the sensation of a weight decrease.

When in free fall (falling with the force of gravity as the only influence) the coaster seat is exerting no force on the passenger. In this instance, the person experiences 0 g's and the sensation of weightlessness.

Inertia

G forces also can be exerted laterally or from side to side. When the coaster goes through a curve, the passengers' bodies want to keep going

forward; thus, they are pinned to the side of the coaster. This can be illustrated by the principle of inertia.

Roller coaster designers can convert lateral g forces into positive g's by "banking a turn." If the carts are tilted inward while going through a curve, the floor, rather than the side of the cart exerts a force on the passenger.

Clothoid Loop

The days of simple roller coasters with a few small hills are long gone. Today's coasters are bigger and scarier than ever, many having loops. Early-day looping coasters had circular loops. At the top of the loop the carts began to slow down and at the end of the loop they were going too fast. The development of the clothoid loop solved this problem.

By decreasing the loop's radius at the top, the speed is increased enough to keep passengers pressed into their seats. At the same time, the radius at the bottom of the loop is increased so that the curve isn't sharp enough to hurt people. The result is a loop that is more elliptical than circular.

Safe?

Speaking of loops often brings up the subject of roller coaster safety. The sight of a body dangling hundreds of feet in the air leads to the question, "How safe are roller coasters?"

Roller coaster designers say, "Safe enough." They have the task of manipulating g forces just enough to make the ride feel dangerous, while using these same forces to keep passengers safely on the coaster.

CLASSROOM ACTIVITIES

In the "*Go Figure*" video, the roller coaster, Batman, at Six Flags St. Louis, is featured. The following problems were developed by the St. Louis Area Physics Teachers Association for Physics Day at the theme park.

1. On a roller coaster, the angle of inclination of the first drop is 50 degrees. A car has a speed of 2 m/s at the top of the incline, which is 50 m long. If it takes 3.8 seconds for the car to travel down the incline, calculate the average acceleration of the car.

Answer: $D = 1/2\ at^2 + vt$

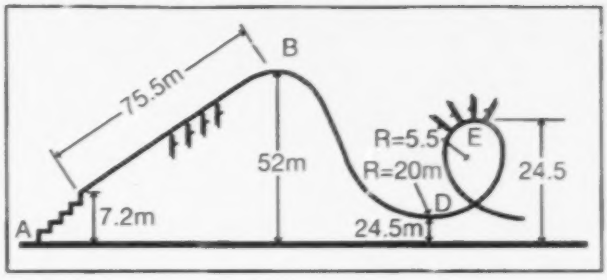
$50\ m = 1/2\ a\ (3.8\ s)^2 + (2\ m/s)\ (3.8\ s)$

$50\ m - 7.6\ m = 1/2\ a\ (3.8\ s)^2$

$a = 42.4\ m\ (2) / (3.8\ s)^2$

$a = 5.9\ m/s$

2. FINDING YOUR TOTAL ENERGY



PE = mgh

Your potential energy at B, the top of the first hill, is the total energy you will have throughout the ride. If we ignore friction, this total energy is the sum of your potential energy and kinetic energy at any given moment. Let potential energy be 0 on the ground and calculate your potential energy at B. This is now your total energy (TE) for the ride.

TE = _____

3. GETTING TO THE TOP--WORK AND POWER

The potential energy at B is a combination of the work you did to get to A and work the coaster did to get you from A to B.

Work = mghA

3a. Find the work you did climbing the stairs, which have a height of 7.2 m.

Work on stairs = _____

3b. Subtract the work you did from the total energy to find the work the coaster did to pull you to the top.

Work AB = _____

Power = work/time

3c. Calculate the power the ride used to get you from A to B.

Power = _____

4. ENERGY AND SPEEDS DOWN AT THE BOTTOM

PE = mgh; TE = PE + KE

During the ride you must account for your total energy as the sum of the potential energy and kinetic energy. At D the potential energy is not 0. (At this point, h = 6.4 m). Fill in the chart below to find your kinetic energy at the bottom.

TE = _____ PE = _____

KE = _____

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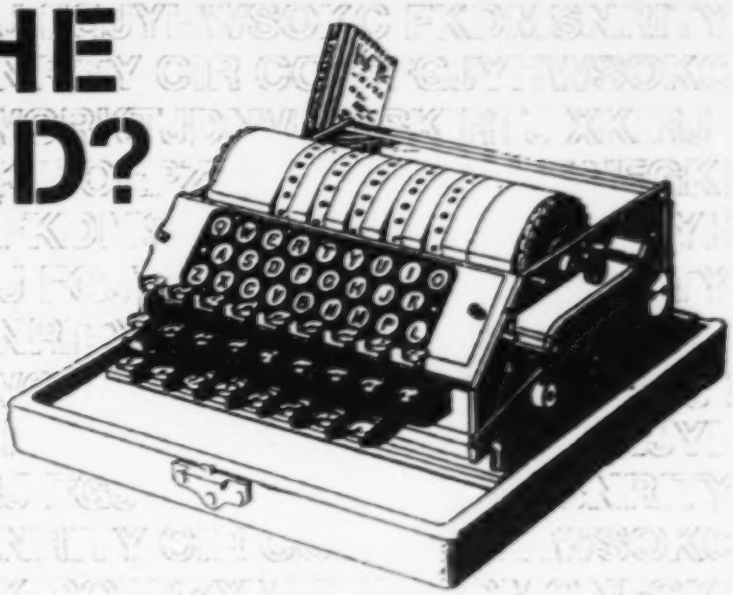
WHAT'S THE PASSWORD?

Cracking the code



Paranoid Man in the *Go Figure* video believes there's "something weird going on" what with all the numbers in our lives–Social Security, driver's license, passports, credit cards, zip codes, area codes, etc. Maybe. But the use of codes isn't weird at all.

Early Jewish writers of sacred texts did it. Ancient Spartan soldiers did it. Cardinal Richlieu did it, as did Thomas Jefferson and James Madison. And the U. S. government, while very secretive about it, does it as well.

What all these disparate folks have in common is using codes and ciphers to conceal and send messages. It's an old art and right in keeping with human nature. As soon as there was some sort of written communication, no doubt, some people wanted to keep a secret just between themselves.

Code or Cipher?

By definition, code refers to using predetermined words, numbers, or symbols to represent other words and phrases. A code is usually impossible to read without the key code book.

A cipher is a method of transposing or substituting the letters of a plain text, or unencrypted, message with other letters or symbols. Ciphers of various types have been devised, but all of them fall into one or both of the two categories, substitution and transposition.

In the Beginning...

Authors of ancient Jewish sacred writings practiced one of the first known ciphers. They reversed the alphabet, using the last letter of the alphabet in place of the first, the next last for the second, and so on. This system of letter substitution, called ATBaSH, has an example in the Bible.

In Jeremiah 25:26, the word "Sheshech" is written for "Babel" (Babylon) using the second and 12th letters from the end of the Hebrew alphabet instead of from the beginning. [In Hebrew there are no vowels between letters, so Babel is "B b l." Since B is the second letter of the alphabet, the second letter from the end (which has the sound "sh") was substituted for "B and b" in Babel. L, the 12th letter of the alphabet, was replaced with the 12th letter from the end of the alphabet (the sound "ch").]

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Letter substitution is one of the oldest and most practiced ciphers, with the Roman emperor Julius Caesar inventing his own. He used a system of advancing each letter four places; for example, in English, "a" would become "d," "b" would be "e," and so on. Today the process is called the Caesar Shift.

Church Secrets

In Renaissance Italy, a small group of devotees to the art of cryptography pioneered the kind of cipher that is now called polyalphabetic substitution.

A country away and a few years later, France's Cardinal Richlieu continued the intrigue. He invented a grille, a card with holes in it, and placed it over a sheet of paper. He'd write a secret message in the holes, then fill in the rest of the paper to look like an innocent letter. Only a person with an identical grille could read the words of the secret message.

Jefferson's Code

A couple of our Founding Fathers got in on the act. A letter written in May 1789 from James Madison to Thomas Jefferson was partially in code. Jefferson had sent the code to Madison four years earlier, and Madison used it to report on the opening of the new federal government. The original letter, in Madison's papers, shows Jefferson's decoded translation written between the lines.

Native American Code Talkers

War and conflict are always an impetus for secrecy, so World Wars I and II were major developments in codes and ciphers. Cipher machines were stock in trade on every side, and code and code breakers were essential. One of the most unusual methods was the use of Native American language by the United States. Choctaw Indian "code talkers" posted in strategic areas simply used their native tongue to send messages during both World Wars I and II. Navajo and Comanche code talkers also were instrumental during World War II. Throughout the two wars, enemy forces were never able to interpret the messages, providing the United States with one of the most secure "codes" of the conflict.

Secrets In the Computer Age

Even with the decline of Communism and the Cold War and the end of the Gulf War, secret messages are still an important issue to governments. In the United States, the field was dominated by the National Security Agency, a federal agency so clandestine that for many years the U.S. government denied the group existed.

The NSA, which gathers intelligence for national security purposes by eavesdropping on overseas telephone calls and cables, did everything in its power to make sure nobody had a code that it couldn't break. However, as computers–the engines of modern cryptography–have increased, so have ever more powerful encryption algorithms (algorithm: a set of rules for solving a problem in a finite number of steps, as for finding the greatest common divisor). And recently, U.S. computer makers announced that they were adopting a new encryption standard so strong that even the NSA couldn't crack it.

All of which has led to a conflict between personal privacy, the rights of business, and the government's need for maintaining security. The Constitution's framers could have had little idea that encryption would one day become the subject of discussions of personal rights and freedoms.

Clipper Chip

The NSA has formulated what is known as the Clipper Chip, a semiconductor device the agency developed and wants installed in every telephone, computer modem and fax machine. The chip combines a powerful encryption algorithm with a "back door"–the cryptographic equivalent of a master key.

Law enforcement agencies argue that they need the master key capability in order to monitor drug runners, terrorists, and spies. Critics denounce the Clipper as a "Big Brother" technology that will strip citizens of what privacy they have left in this age of computers and easy access to once-personal information.

Minding Your Ps and Qs: Creating a Code or Cipher

Substitution, transposition, mono- and polyalphabetic and digraphic systems, Vigenere tables—these are just a few of the choices a budding cryptographer can make in sending a secret message. Most all use math and/or geometry in setting up the cipher, and numbers can be used in some substitution schemes.

Transposition Ciphers

Transposition ciphers involve the use of a geometric design, such as a square or rectangle. The cryptographer inscribes the plain text letters by one route in the design, then transcribes them by another route to form the cipher text. In columnar transposition, the cryptographer chooses a numerical key and writes the plain text letters under it, then takes the columns of letters in key-number order to form the cipher text.

Substitution

Substitution systems can also vary. In simple substitution ciphers, a particular letter or symbol is substituted for each letter in the plain text message. The letters are left in their normal order, usually with normal word divisions. These ciphers can be "decoded" by recognizing the occurrence of a set of normal letter frequencies attached to the wrong letters, for example the TH in the, there, they, these, etc.

In multiple substitution (polyalphabetic) ciphers, a keyword or number is used. The first message letter might be enciphered by adding the numerical value of the first letter of the keyword; the second message letter is enciphered in the same way, and so on, repeating the keyword as often as needed to encipher the whole message. Thus, to encipher the word TODAY by the code word DIG, T becomes W (since D is the fourth letter of the alphabet, we move over four letters from T, counting T as the first letter), O becomes W (since I is the ninth letter of the alphabet, we move over nine letters from O, and so on through the message).

Computer Ciphers

Then there are the computer ciphers. The continual advancement in computer technology has allowed some very strong (that is, tough to translate) ciphers to be created. One method is to treat the characters in the message as digits in a very large number, raise that number to a power, divide it by another very large number and output the remainder. It would be nearly impossible to break the cipher randomly.

Besides the applications in math and geometry, cryptography could have some very practical applications for teachers. How many students could decipher a test-key written code?

CLASSROOM ACTIVITIES

1. Using the substitution system, develop a cipher for the name of your school, city and state, etc., for your class to unravel. For example, Lincoln Middle School would become BFCGSBC RFZZBA JGMSSB (where B=l; F=i; C=n; G=c; S=o; R=m; Z=d; A=e; J=s; M=h) or numerically, it would become 0123402 516607 839440 (where 0=l; 1=i; 2=n; 3=c; 4=o; 5=m; 6=d; 7=e; 8=s; 9=h).
(Note: An excellent source for ciphers can be found in most daily newspapers on the crossword-puzzle page.)

2. Divide the class into teams and challenge them to be first at correctly enciphering the word "mathematics" using a code word of your choosing or by using the code word DIG.
The answer using DIG as the code word:
PIZKMSDBOFA

(Solution: M becomes P (since D is the 4th letter of the alphabet, we move over 4 letters from M); A becomes I (I is the 9th letter of the alphabet; move over 9 letters from A); T becomes Z (G is the 7th letter of the alphabet; move over 7 letters from T); H becomes K (back to D, 4th letter of the alphabet; move over 4 letters from H); etc.

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Saw Me a Tune

Music and math have been associated throughout history. The Greek mathematician Pythagoras used numbers to model everything in the physical world, including music. He noted music's regularities, including tempos and scales. And the great composer, Johann Sebastian Bach, reportedly sometimes treated the composition of music as exercises in solving mathematical puzzles.

Sound of Math

Today, the language of mathematics is used to represent most musical ideas. Musicians know what notes to play and how to perform a certain piece of music thanks to time signatures, note representation, and scale progressions. Just as important, math is used to analyze music and other sounds.

Music can be thought of as organized sound. Sound begins when some physical body, in contact with the air, vibrates. The surrounding atmosphere then begins to vibrate, causing periodic variations in atmospheric pressure, which are called sound waves. These waves are much like the ripples seen when a rock is dropped into a pond. The bones and skin of the ear are pushed by these rippling waves, and nerves send a message to the brain where the sound is interpreted. Because sound can't be seen, scientists make graphical representations on paper. Thus, sound becomes math.

A simple way to produce a sound wave is with a tuning fork. When struck, a tuning fork vibrates and produces an audible tone. If these vibrations are graphed, a sine curve is formed. By studying this curve, the frequency, loudness, and quality of a sound can be determined.

Harmony & Noise

Each cycle of a sound wave includes one compression and one rarefaction. The frequency is the number of cycles the sound wave completes per second. Frequency, measured in Hertz, is an important element in music because sounds that have frequencies with simple mathematical relationships are usually interpreted by people as being harmonious.

Not all sounds are pleasant ones. For instance, striking two stones together produces a vibration, but the vibration is not harmonious. That's because sound vibrates from the stones in all three dimensions. The frequencies of the sound wave are of great variety and have nothing in common. Such sound usually is called noise.

Making Waves

Musical instruments produce their own waves, which, in most cases, are set up in one dimension only. Because the waves travel in one dimension, the frequencies are related mathematically, and therefore, we hear a harmonious sound. For example, a guitar produces vibrations along its strings. The sound of a drum is less harmonious than other instruments because it vibrates in two dimensions.

When a musician places a finger on the string of an instrument, the wavelength is shortened, increasing the frequency. The frequency of a tone is inversely related to the length of its vibrating string. Pythagoras noted that musical tones are related to the length of the string by exact ratios. Certain simple ratios give the most harmonious intervals, according to Pythagoras, and that is the basis for the musical scales we use today. By holding down a string at its midpoint and plucking a free half, the tone produced is exactly one octave above the tone of the entire string.

A sine curve also can illustrate the volume or intensity of a sound. The higher the volume, the higher will be the amplitude of the sound wave. However, the frequency is not affected by the volume.

Musical Glasses

The same mathematical ideas can be applied to all musical instruments to help designers produce the most enjoyable music possible. Furthermore, almost any object can be turned into a musical instrument with a little help from mathematics. For example, a set of drinking glasses filled with different amounts of water will create various tones if hit lightly with a wooden spoon. By using different amounts of water, the glasses' wavelengths are changed much like when a guitar string is depressed.

Musical Saws

Sometimes ordinary musical instruments are made from not-so-ordinary objects. Who would have thought that a handsaw could make music? Someone did. In fact, musical saws date back to the 12th century. Today's musical saw has many devoted players as well as fans.

These days, even the most unusual musical instruments can be simulated in a studio. With the advance of electronic and computer-generated sound in the music industry, mathematics is

becoming even more useful to musicians.

So when the next rock group sensation comes along, they'll have the ability to produce better-quality music, thanks to the blurred line between mathematician and musician.

Classroom Activities

1. Sound waves travel on a guitar's string after the string is plucked. The length of the string between its two fixed ends is 0.628 meter, and the string's mass is 0.208 gram. The string is under a tension of 226 Newtons. Find the velocity (v) of the wave on the string.

Solution: The tension F, the mass m, and the length L are known. Use the formula

$$V = \sqrt{\frac{F}{mL}}$$

to solve the problem.

2. The same guitar string is depressed by the musician, making the new length .5 m. Find the new velocity using the same formula as above.

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GUESS WHAT!

Estimation Pros

In everyday life, *estimations* have to be made in countless situations. Is there time for a soft drink before the next class? Can I stop for a sandwich and still make it to the movie on time? If I watch TV tonight, is there time to do that math assignment in the morning? The *Go Figure* video mentions a more complex use of estimating—determining the population of our nation.

Census Sense

Estimation is used in some businesses, such as the fast food industry, because the number being sought will not be known until after the fact. However, in other areas, estimation is used because the number being sought is either too large to count or immeasurable for some reason. That's why sampling is often used in census taking.

In the United States, for example, it would be impossible to go door to door and count every person. The U.S. Census Bureau now sends census questionnaires to every household in the country. If every household returned the form, the census would be complete. Yet, many choose not to participate. Therefore, the Census Bureau must account for those who do not return the forms by making estimations.

After a flurry of controversy over accuracy of the 1990 population count, the Census Bureau is vowing to make Census 2000 the most accurate in history. To accomplish this, citizens will be given multiple opportunities to respond, including an initial mailing, a reminder card, forms available at public locations, and a toll-free telephone number. The Census Bureau also is planning an advertising campaign, and census organizers will contact state, local and tribal governments for help in boosting participation.

The sampling plan requires direct contact with 90 percent of all housing units. An accurate listing of all addresses in every census tract will be key in determining if 90 percent has responded. A random sample of housing will then be selected that will represent the number and characteristics of those who did not respond.

An accurate census is important since population figures determine the number of governmental representatives an area will have.

Who's Watching What?

Estimation also is often used to count how many people are doing what. How else could it be determined how many people watched the last episode of "Seinfeld?" Going door to door asking, "What TV programs did you watch last night?" would be costly and time-consuming. Thus, TV ratings are reached by estimation.

One method used to collect viewing data is the diary. Sample families across the country are asked to keep diaries detailing which TV programs

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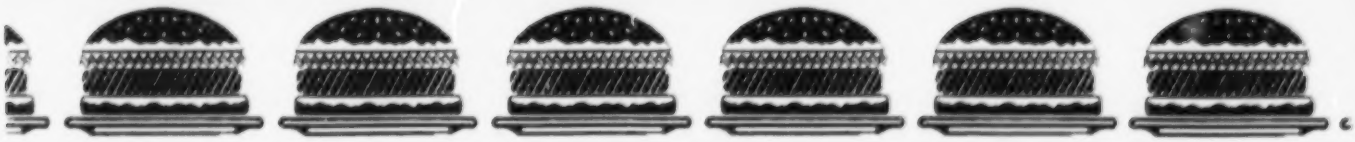


they watch. At the end of the week, the diaries are mailed to a rating company where they are calculated and compared. The resulting numbers determine the placement of billions of dollars in TV advertising each year, proof that estimating plays a critical role in the success of some businesses.

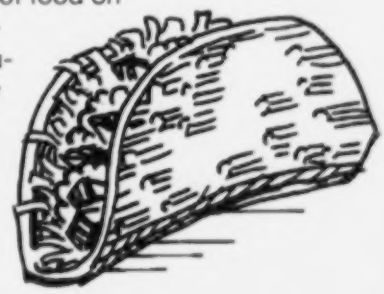
Estimation gives insight into numbers that normally would not be accessible, and many numbers we depend on every day, from unemployment rates to the current price of gasoline, are derived from estimations.

Grocery Shopping

A routine trip to the grocery store, for example, definitely can illustrate the value of estimating. Which groceries must be bought now, and which ones can be purchased later? Let's see; the milk jug is empty, only a few crumbs linger in the cereal box, and the grapes have turned to raisins. Two slices of bread remain; unfortunately, both have turned green. The ketchup bottle has been flip-flopped, pounded, and shaken until not an ounce is left. Also there are no eggs, butter, soda, and paprika. All these to buy and only X dollars to spend.

Actually, the checkbook is still at home next to the forgotten grocery list. An ATM card and a vague memory of the checking balance will have to suffice. This calls for some major estimating.

Grocery shopping, whether for one person or a large family, can be a real task, but just imagine the *estimation skills* it takes to feed hundreds of people and keep plenty of food on hand. That's what managers of fast food restaurants across the country have to do on a daily basis.



For most restaurants, a delivery truck runs two to three times per week. Managers must order enough food to last until the next delivery. At the same time, over-ordering creates problems because the food can spoil before it is used.

Predicting by the Past

This is a difficult balancing act in an industry where sales vary depending on a variety of factors including the weather, day of the week and competitors. Therefore, in the fast food industry, the future is predicted by looking at the past.

Restaurant managers often use sales projections based on data from previous years' sales in a particular month. However, a store's overall sales may have increased or decreased since last year, so managers also must look at sales for last month and last week.

How Many Burgers?

Having enough food in stock is only half the battle. Restaurant managers also must decide how much food to prepare throughout the day. Obviously, more food is required for lunch and dinner rushes than at any other time. For this reason, most restaurants keep a record of hourly sales for every day of the week. Managers can analyze a particular hour's sales for the past several weeks and get an average. They then use preestablished formulas to decide how much food needs to be prepared for that hour.

Classroom Activity

You are working at a Mexican fast food restaurant. It is 2 p.m. Saturday. Looking through the record book, you find that on Saturdays between 2 p.m. and 3 p.m., sales for the last four weeks have been $168, $195, $129, and $140. The company handbook tells you that two pounds of refried beans should be enough for $55 in sales and that beans can only be cooked in two-pound increments. How many pounds of refried beans should you have on hand? (Hint: It's better to have too much food than not enough to feed your customers.)

Solution: First, average the last four Saturdays' sales. ($158). Next, divide $158 by $55 to determine how many two-pound sales you need to prepare for (2.87). Then round off 2.87 (3 two-pound sales). ANSWER: 3 x 2 = 6. You should have 6 pounds on hand.

Sources

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